Super-resolution via spatial mode demultiplexing and its applicability to observational astronomy

Study Reference Number: 17-1501
Type of activity: Standard Study (30k€)

Project Summary

Objective

This project will experimentally determine the domain of validity of recent methods for super-resolution using spatial mode demultiplexing.

Target university partner competences

Experimental optics, optical imaging, spatial mode demultiplexing (e.g. via spatial light modulation, fiber optics…)

ACT provided competences

Computational optics (simulations), quantum optics, observational astronomy

Keywords

Super-resolution, quantum information, indirect imaging, spatial mode demultiplexing, observational astronomy, binary stars, exoplanet detection

Study Objective

This study will explore the concept of indirect super-resolution imaging based on spatial mode demultiplexing [1]–[4]. The expected outcome of the study is an experimental assessment of the parameter ranges in which the proposed method can perform better than conventional imaging. Based on this, the applicability of the method to observational astronomy should be studied, with reference to binary/multiple star systems and exoplanet detection.

Background and Study Motivation

According to the widely accepted Rayleigh criterion, in order to resolve two distant point sources, their angular separation $\Delta\theta$ must be larger than $1.22\ \lambda/D$, where λ is the wavelength of light and D is the telescope's aperture diameter. To achieve higher resolutions, telescope systems with ever larger primary mirrors are therefore being designed, making them extremely expensive and technologically challenging to build.

The problem of limited resolution has been recently revisited and studied in a number of papers from a more statistical perspective – through the so called Fisher information [1]–[7].

When resolving two point sources positioned at angles θ_1 and θ_2, our objective is to estimate their separation $\Delta\theta = \theta_2 - \theta_1$. The statistics of the appropriate quantum measurement (e.g. photon counting) carries information about $\Delta\theta$. In particular, the Fisher information $\mathcal{F}$ is a statistical quantity that assesses the information gained by performing a measurement on a physical system [5], [8]; it is a mathematical measure of the sensitivity of an observable quantity (the distribution of photons across pixels/channels) to changes in its underlying parameters (separation $\Delta\theta$).

By calculating the Fisher information for the case of conventional image-plane photon counting, it has been shown that $\mathcal{F}$ drops to zero as the separation of the point sources goes to zero – a phenomenon that has been dubbed Rayleigh's curse [1]. Nevertheless, by changing the measurement scheme, it is possible to have a non-zero Fisher information for every source separation $\Delta\theta$, rendering the Rayleigh criterion moot [1]–[4]. Limitations on our ability to resolve two incoherent point sources is therefore not a fundamental constraint, but rather a consequence of the most commonly applied measurement technique - wherein a pixelated detector is used to measure the light intensity distribution in the image plane of the system.

One improved measurement scheme uses spatial light demultiplexing, in which the electric field in the image plane is decomposed into a set of spatially independent modes - for example Hermite-Gaussian (also TEM) modes. By measuring the corresponding signal in a number of lower order modes, it is then possible to extract information about the separation and brightness of the two point sources, even for separations below the conventional resolution limit. By decomposing the image field into a symmetric and an antisymmetric mode, the method has already been experimentally tested and proven to work for two equally bright incoherent point sources [2].

This method is believed to be beneficial for particular applications in observational astronomy. For example, two stars in a distant binary system that would appear as a single spot on a directly measured image could be properly resolved using this approach. It might even be possible to generalize the measurement technique to resolve multiple star systems [9]. Another application could be towards the detection of exoplanets, where the huge difference in brightness between the host star and its planet imposes an additional challenge for direct imaging, while large differences in brightness may be less problematic for detection via spatial light demultiplexing.

The partner university is expected to experimentally determine the domain of validity (in an appropriate parametrization of the model) of the spatial mode demultiplexing method for super-resolution. There are several possible ways to perform spatial mode demultiplexing. One option is coupling the image-plane field into a multimode fiber with its propagation modes corresponding to the desired decomposition modes and then separating the signal in each mode by using a grating coupler or a series of single-mode fibers coupled to the multimode fiber [1]. Other option is to project the image field onto different modes via digital holography using a spatial light modulator (SLM) [2]. We believe the latter is a more suitable approach of the two, since an SLM can be easily reprogrammed to project onto a different set of modes or for the use at a different wavelength. The partner university is, however, encouraged to propose another spatial mode demultiplexing technique, but the final choice should be made in agreement with the ACT.

Proposed Methodology

The study would be performed in close cooperation with the ACT. The following workflow is proposed, although the university partner is encouraged to suggest alternative methods alongside some justification of why they are more promising:

1. Choice of theoretical model. In consultation with the ACT, an appropriate theoretical model is adopted, as well as a parametrization of the problem.

2. Simulation of model. Also in consultation with the ACT. Investigation of parameter space by the means of numerical simulation in order to identify parameter ranges in which the proposed method is expected to perform better than conventional imaging.
3. Experimental analysis. An experimental realization by the university for the case of two unequally bright incoherent point sources. This should identify the sources of error and determine the domain of validity - in parameter space - of the method.
4. Application to astronomy. Together with the ACT, an analysis of the results is performed in order to evaluate the potential of the proposed measurement scheme for observational astronomy, particularly for binary star and exoplanet detection.
5. Extension of the model. A possible extension to more complex objects that are described with more parameters (e.g. more than two sources, extended sources,…) and its experimental realization.

ESA/ACT Contribution

The ACT will contribute in several ways, depending on the needs and available expertise. We can provide a starting theoretical model for the case of two unequally bright incoherent point sources which could later be improved. Additionally, the ACT can offer expertise in the following areas:

- Numerical simulations
- Quantum physics/optics
- Observational astronomy

Bibliography

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